Int. J. Multiphase Flow Vol. 5, pp. 427–436
Pergamon/Elsevier, 1979. Printed in Great Britain

INFLUENCE OF LARGE EDDIES ON THE SUSPENSION OF SOLID PARTICLES

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(*Received* 25 *October* 1978; *in revised form* 19 *September* 1979)

Abstract—The phenomena of solid particles suspensions, in a turbulent flow, can more conveniently be described by stochastic models than by diffusion models, particularly in the case of relatively coarse particles.

The fundamental difficulties of using such models are principally due to the difficulty of performing direct measurements of probabilities, because the number of observations (or tests) necessary to obtain physically representative values is important (theoretically infinite).

We have used such a model to describe the movement of spheres in an inclinable pipe.

To do so, we have identified the movement through a Markov process which permits us to show that we can characterize it by the limit distribution for passage probabilities in a cross section. We have used a special system of close-circuit television to measure it, doing a sufficiently large number of observations for the measurements to be significant.

In the case of a vertical pipe, the phenomena is one-dimensional. By using the model stochastic displacement, we obtain a differential equation which it is possible to integrate by assuming an obviously constant radial dispersion. The interpretation of limit distributions for passage probabilities and visual observations of particles movement in the pipe have caused us to conclude that the mean displacment is due, on one hand, to a radial acceleration bounded to a stochastic rotation of the flow and, on the other hand, to the effect of the mean velocity gradient. The experimental results show that the radial dispersion is a function of the relative dimension of particles with respect to the macroscale of the turbulence.

In the case of an inclined pipe, a two-dimensional stochastic model of the displacement is possible, but the integration of the equation is quite complicated and may be done numerically. We have prefered a two-dimensional simulation model. The results of the simulations permit us to obtain a limit repartition of passage probabilities, the moments of which we have compared with those that we have measured. These comparisons show that the model obviously represents the phenomena when the pipe is horizontal or very slightly inclined but differs in the near vertical case. This is due to the simplicity of the model in which we neglect the radial acceleration we have considered previously and the effect of which is negligible in comparison with gravity when the pipe is inclined.

The interpretation of the measurements by comparison of moments with the two-dimensional model shows that the angular dispersion of solid particles is essentially due to big eddies and that the particle diameters are not essential parameters in this case.

By associating this conclusion with that obtained previously concerning the radial dispersion, it seems that the eddies bigger than the macroscale of turbulence may be of capital importance in the dispersion of solid particles and that it will be of practical interest to characterize them as a function of a mean parameter of the flow.

The study of the movement of sufficiently large particles seems to be a method which is able to give this result.

1. DETERMINISTIC MODELS AND STOCHASTIC MODELS

To describe accurately the phenomena of solid particles suspension in a turbulent flow, it is necessary to do a local and instantaneous description of the flow, and of the solid particles which can be of different shapes and dimensions. A detailed analysis shows that one of the principal suspension parameters is the turbulent agitation which is characterized by a large number of stochastic parameters. In these conditions, we can try to find a relation between the mean statistical values of parameters, in two different ways:

—We suppose that each parameter is determined by a mean statistical value and we establish a model joining these different values; we have used a deterministic model.

—We establish a model joining the local and instantaneous values of each parameter considered as stochastic variables and we take the statistical mean of the obtained relation: we have used a stochastic model.

Let us consider particles in suspension in a turbulent flow. If we suppose that the statistical mean of the solid particle concentration $\bar{c}$, is a passive scalar, we can describe the suspension evolution by the classical diffusion equation (Hinze 1959; Fortier 1967).

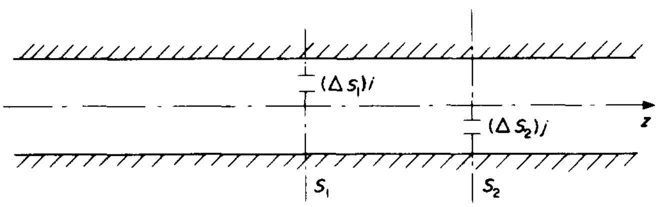
$$\frac{\partial \bar{c}}{\partial t} + \partial_i\{\bar{U}_s^i \cdot \bar{c}\} = \partial_i\{(\epsilon_s)_j^i \partial^j \bar{c}\}$$

where ∂_i is the operator $\partial/\partial x^i$; $\bar{U}_s^i$ is the statistical mean instantaneous velocity of solid particles; and $(\epsilon_s)_j^i$ is the diffusion tensor of solid particles.

The direct measurement of the different tensor is impossible to do; we can obtain it by adjusting experimental results on the next equation. We have tried to obtain a description of solid particle movement by direct measurement of the characteristic parameter of their stochastic displacements.

2. STOCHASTIC DISPLACEMENT OF A PARTICLE IN A PIPE

2.1 *Identification to a Markov process*



Let z be the longitudinal axis of the pipe. We define, in each cross section s, elementary areas $(\Delta s)_i$. Between two cross sections s and s_2 we can define the probability p_{ij} that a particle will pass through $(\Delta s_2)_j$ of s_2, if it has passed through $(\Delta s_1)_i$ of s_1. The elements p_{ij} define a passage matrice P of a Markov process if they only depend on the relative position of $(\Delta s_2)_j$ and $(\Delta s_1)_i$, and not on the past history of the particle. Such an hypothesis is physically acceptable if we choose the sections s_2 and s_1 far enough apart so that the turbulent interactions of flow on the solid particles between s_1 and s_2 are stochastically independant of those on particles before section s_1.

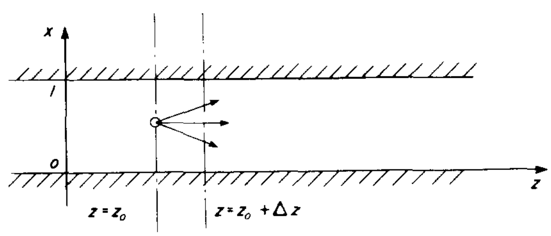
In these conditions, the stochastic vector $\mathbf{X}$, which defines the particle position in a cross section s, follows a Markov process. It is physically clear that this process is:

—Homogeneous, if we consider a steady flow and if the pipe characteristics are independant of z.

—Irreducible, since the variable $\mathbf{X}$ can take all possible values.

Since the passage probability distribution, characterised by the vector p, tends to a limit π when $z \to \infty$, we say that the equilibrium distribution is reached (Cox & Miller 1970). π can be characteristic of the Markov process.

2.2 *Monodimensionnal displacement model*



Let us consider a particle located at the transversal position x in the section $z = z_0$, and let $\alpha(x)$ be the probability for this particle to be in the section $z_0 + \Delta z$ in the position $(x + \Delta x)$, and $\beta(x)$ be the probability for it to be in the section $z_0 + \Delta z$, in the position $(x - \Delta x)$.

If we indicate by $p(x, z)\Delta x$, the probability that a particle is located between x and $x+\Delta x$, we have:

$$p(x, z) = \alpha(x-\Delta x)p(x-\Delta x, z-\Delta z) + [1-\alpha(x)-\beta(x)]p(x, z-\Delta z) \\ + \beta(x+\Delta x)p(x+\Delta x, z-\Delta z).$$

If we suppose that the functions $p(x, z)$, $\alpha(x)$, $\beta(x)$ are developable, in series, we obtain:

$$\frac{\partial p}{\partial z} = \left\{p(x, z)\left[\frac{\mathrm{d}\beta(x)}{\mathrm{d}x} - \frac{\mathrm{d}\alpha(x)}{\mathrm{d}x}\right] + \frac{\partial p}{\partial x}[\beta(x)-\alpha(x)]\right\}\frac{\Delta x}{\Delta z} + \left\{\frac{\partial p}{\partial x}\left[\frac{\mathrm{d}\alpha(x)}{\mathrm{d}x} + \frac{\mathrm{d}\beta(x)}{\mathrm{d}x}\right]\right. \\ \left. + \frac{1}{2}\frac{\partial^2 p}{\partial x^2}[\alpha(x)+\beta(x)]\frac{1}{2}p(x, z)\left[\frac{\mathrm{d}^2\alpha(x)}{\mathrm{d}x^2} + \frac{\mathrm{d}^2\beta(x)}{\mathrm{d}x^2}\right]\right\}\frac{\Delta x^2}{\Delta z}.$$

Noting that $\{\alpha(x)-\beta(x)\}\Delta x$ is the mean displacement between the two sections distant of Δz, and $-\{[\alpha(x)+\beta(x)]-[\alpha(x)-\beta(x)]^2\}\Delta x^2$ is the variance of this displacement, and letting

$$\mu(x) = \lim_{\Delta x, \Delta z \to 0} \{\alpha(x)-\beta(x)\}\frac{\Delta x}{\Delta z}$$

$$\sigma(x) = \lim_{\Delta x, \Delta z \to 0} \{[\alpha(x)+\beta(x)]-[\alpha(x)-\beta(x)]^2\}\frac{\Delta x^2}{\Delta z}$$

we can obtain the following equation:

$$\frac{\partial p}{\partial z} = \frac{\partial^2}{\partial x^2}\left\{\frac{1}{2}p(x, z)\cdot\sigma(x)\right\} - \frac{\partial}{\partial x}\{p(x, z)\cdot\mu(x)\}.$$

If $f(x)$ is the limit of $p(x, z)$ when $z\to\infty$, we obtain:

$$f(x)\cdot\mu(x) = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}x}\{f(x)\cdot\sigma(x)\} + A.$$

The constant A can be determined by the limit conditions, by writing that the probability of a particle being in the position $x>1$ or $x<0$ is 0; we show that, in this case, $A=0$.

The equilibrium distribution is given by the differential equation:

$$f(x)\cdot\mu(x) = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}x}\{f(x)\sigma(x)\}. \qquad [1]$$

2.3 *Two-dimensionnal displacement model*

If we consider that the stochastic variable **X** defining the particle position in a cross section is a two-dimensional vector and if we let $\mathbf{X}_i$ be its components then, in the same way, we can define: $\boldsymbol{M}$ the vector of components, M_i as the mean displacement between two sections z and $z+\Delta z$ such that:

$$M_i(x) = \lim_{\Delta z\to 0} E\{[\mathbf{X}_i(z+\Delta z)+\mathbf{X}_i(z)]/\mathbf{X}(z)=x\}\cdot\frac{1}{\Delta z}$$

where $E\{a\}$ is the mean of a, and Σ, the tensor of components σ_{ij}, such that:

$$\sigma_{ij}(x) = \frac{1}{\Delta z}\cdot\lim_{\Delta z\to 0} C\{[\mathbf{X}_i(z+\Delta z)-\mathbf{X}_i(z)], [\mathbf{X}_j(z+\Delta z)-\mathbf{X}_j(z)]/\mathbf{X}(z)=x\}$$

where $C\{a, b\}$ is the covariance of a and b.

We obtain an equation similar to [1]:

$$\frac{1}{2}\partial_i\partial_j\{\sigma^{ij}(x)\cdot p(x,z)\} - \partial_i\{M^i(x)\cdot p(x,z)\} = \frac{\partial p}{\partial z}.$$

The equilibrium distribution $f(x)$ which is, in polar coordinates, a function of r and θ, is a solution of:

$$\frac{1}{2}\left[\partial_{rr}(r\sigma^{rr}f) + \left(\partial_{r\theta} + \frac{1}{r}\partial_\theta\right)\{(\sigma^{r\theta}+\sigma^{\theta r})f\} - \left(\partial_r - \frac{1}{r}\partial_{\theta\theta}\right)\{\sigma^{\theta\theta}f\}\right] - \partial_r\{rM^rf\} - \partial_\theta\{M^\theta f\} = 0. \quad [2]$$

3. MEASUREMENTS OF PASSAGE PROBABILITIES REPARTITION IN A CROSS SECTION

3.1 *Description of the detection system*

An estimation of the passage probabilities distribution can be obtained:

—By dividing each cross section in elements $(\Delta s)_i$, located by the vector x_i.

—*By releasing* N times, one particle into the flow and counting the number of times n_i that a particle passes through $(\Delta s)_i$.

So, we obtain:

$$f(x_i) = \frac{n_i/N}{(\Delta s)_i}.$$

Theoretically, we should have $N\to\infty$ and $(\Delta s)_i \to 0$. Practically, in order to have an acceptable precision, using the Moivre–Laplace law, we can show that it is necessary to have $N \sim 1000$.

It is necessary to use a suitable detection system. The principle of the system used is the following (figure 1):

"We light the cross section s, maintaining the test of pipe in darkness. A particle moving in the flow is only visible when it passes through s. The picture of this passage is formed on a television camera, by a periscopic system. The analysis of this picture is realised by a special apparatus (Moya Anica 1972; Alquier 1975; Alquier & Gruat 1978) which permits the particle passage to be obtained immediately. The measured values are registered on a writing recorder and analysed after each experiment".

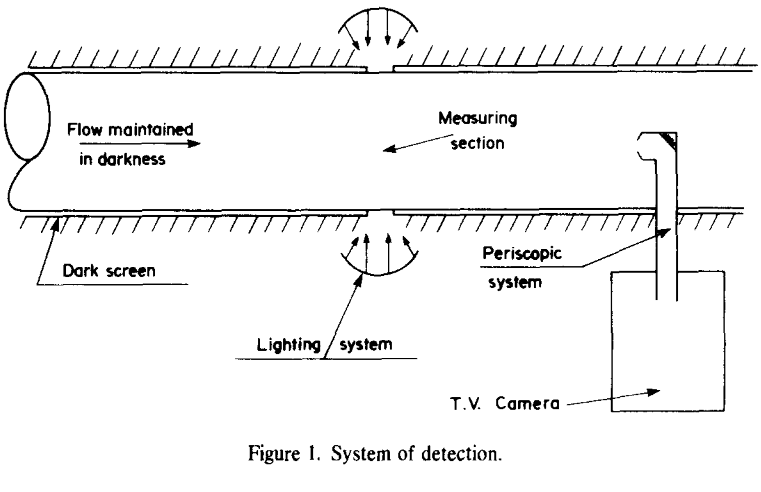


Figure 1. System of detection.

Table 1. Characteristics of solid spheres

diameter in mm.	5	4	3	2	3
density in kg/m3	1375	1313	1437	1593	804
fall velocity in cm/s	20,7	18,3	15,1	·. 11,2	- 10,9

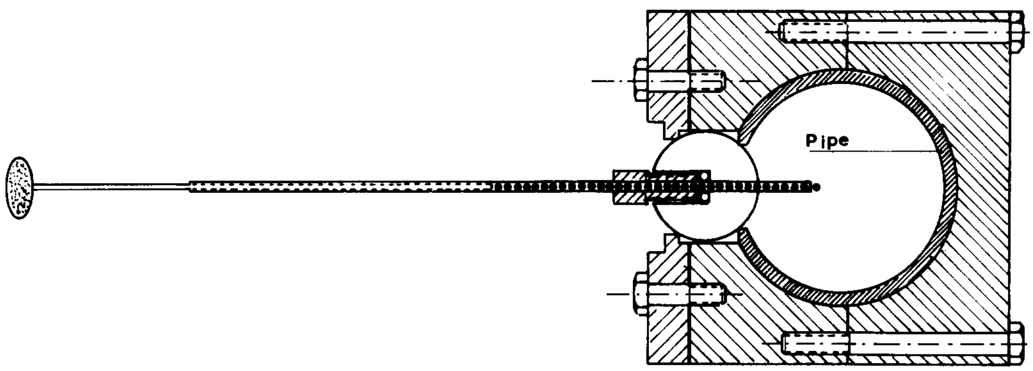


Figure 2. System of particles injection.

3.2 *Experimental conditions*

The experimental installation is an inclinable pipe 4.50 m long and 80 mm diameter, with a grid in the entry in order to obtain a fully developed flow as quickly as possible.

For each test, we inject approx. 1000 spherical particles, one by one, the characteristic of which are given by table 1. The injection system is given on figure 2. Experiments have been done with different injection points and it has been verified that the results are not influenced by the position of injection point.

We have done tests for differents discharges and different values of the angle of slope α with respect to the horizontal (see table 2).

3.3 *Experimental observations*

By looking at the particle displacements, we can see that they have small stochastic movements in the radial and angular directions which show the effects of turbulence, and larger scale movements (of the same order of magnitude as the pipe radius) consisting, with small velocities of alternate, and apparently stochastic, rotations round the axis of the pipe. These movements correspond to a stochastic twisting flow which can be simulated by an eddy structure with a scale of same order of magnitude as the pipe radius.

In these conditions, it appears that the radial dispersion is essentially due to the fine structures and must be a function of the particle dimension, since the angular dispersion is essentially due to the big structures and depends little on particle size.

4. RADIAL DISPERSION STUDY

In the case of a vertical pipe, there is an axial symmetry so that the passage probability depends only on the distance to the pipe axis and can be characterised by the repartition function $Q(r)$, where r is a non-dimensional variable varying between 0 and 1:

$$r = \frac{R}{(D-d)/2}$$

Table 2.

Q en l/s \ α	90	82.5	75	60	30	0	V_{mean} in m / s	$Re = \frac{V_m D}{\nu}$
7.1	X						1.44	1.14×10^5
5.7	X	X	X	X	X	X	1.16	9.19×10^4
4.6	X						0,938	7.41×10^4
3.1	X	X	X	X	X	X	0.632	5×10^4
2.7	X						0.551	4.35×10^4
2	X						0.408	3.22×10^4
1.5	X						0.306	2.42×10^4

where R is the radial coordinate; D is the pipe diameter; and d is the sphere diameter.

$Q(r)$ is the probability that the passage radius of a particle is bigger than r. We can use the unidimensional model, the equilibrium distribution of which is given by [1].

$\sigma(r)$ which is due to turbulent radial fluctuations can, to a first approximation, be taken to be a constant; $\mu(r)$ is due (i) to the big eddies which induce a rotation round the axis of the pipe and so a radial acceleration field, and (ii) to the effect of wall which is perceptible since the particle is sufficiently near the wall, i.e. since $r > r_p$.

So, we have put

$$\begin{cases} 2\mu(r)/\sigma(r) = k_0 + k_1(r - r_p) & \text{if} \quad r > r_p \\ 2\mu(r)/\sigma(r) = k_0 & \text{if} \quad r < r_p \end{cases} \qquad [3]$$

where k_0, k_1, r_p are constants. The equation [1] gives:

$$f(r) \cdot \{2\mu(r)/\sigma(r)\} = \frac{df}{dr}$$

where

$$\begin{cases} f(r) = f_0\, e^{k_0 r + (1/2) k_1 (r - r_p)^2} & \text{if} \quad r > r_p \\ f(r) = f_0\, e^{k_0 r} & \text{if} \quad r < r_p \end{cases}$$

and

$$\begin{cases} Q(r) = \dfrac{1}{A} \displaystyle\int_1^r r'\, e^{k_0 r' + k_1 (r' - r_p)^2}\, dr' & \text{if} \quad r > r_p \\ Q(r) = \dfrac{1}{A} \displaystyle\int_{r_p}^r r'\, e^{k_0 r'}\, dr' & \text{if} \quad r < r_p \end{cases}$$

with

$$A = \int_0^{r_p} r'\, e^{k_0 r'}\, dr' + \int_{r_p}^1 r' \{e^{k_0 r' + (1/2) k_1 (r' - r_p)^2}\}\, dr'.$$

By using the mean square criterion, we have determined the k_0, k_1, r_p values, corresponding to each test. The results for k_0 are given on the figure 3 for different particles and different

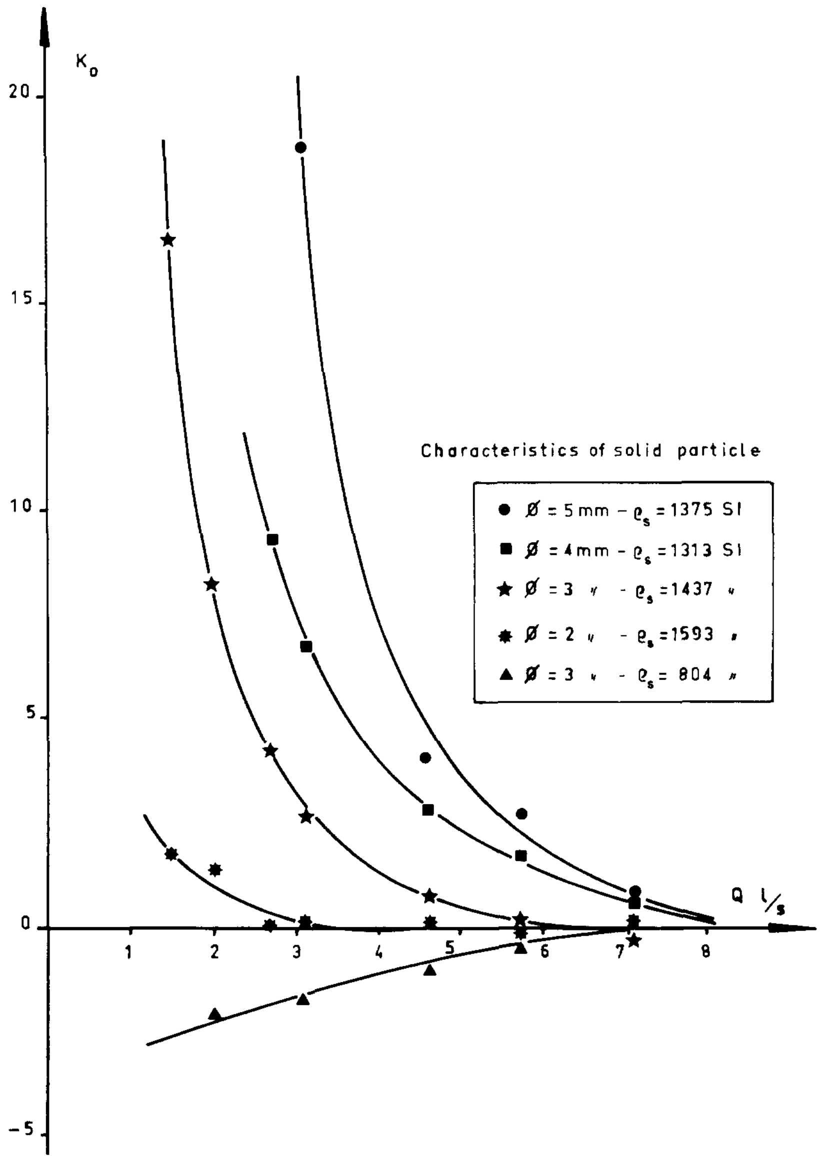


Figure 3. Radial dispersion study-coefficient K_0 of [3].

discharges. We can notice that for the lighter than water particle, k_0 is negative, which corresponds to a centripetal acceleration, and $k_0 \to 0$ when discharge increases, i.e. when σ become sufficiently large.

The mean radial displacement $\mu(r)$ varies as the relative value of transverse velocity due to radial acceleration on the mean longitudinal velocity. Dimensional analysis shows that (Alquier 1977):

$$\mu \sim \frac{\rho_s - \rho}{\rho} \frac{d^2}{D^2} \mathrm{Re}$$

so that

$$k_0 \cdot \left[1 \Big/ \left\{ \frac{\rho_s - \rho}{\rho} \cdot \frac{d^2}{D^2} \cdot \mathrm{Re} \right\} \right] \sim 1/\sigma \qquad \text{(Komasaka } et\ al. \text{ 1974)}.$$

The macrosccale of turbulence Λ varies to a first approximation as ReD so that the

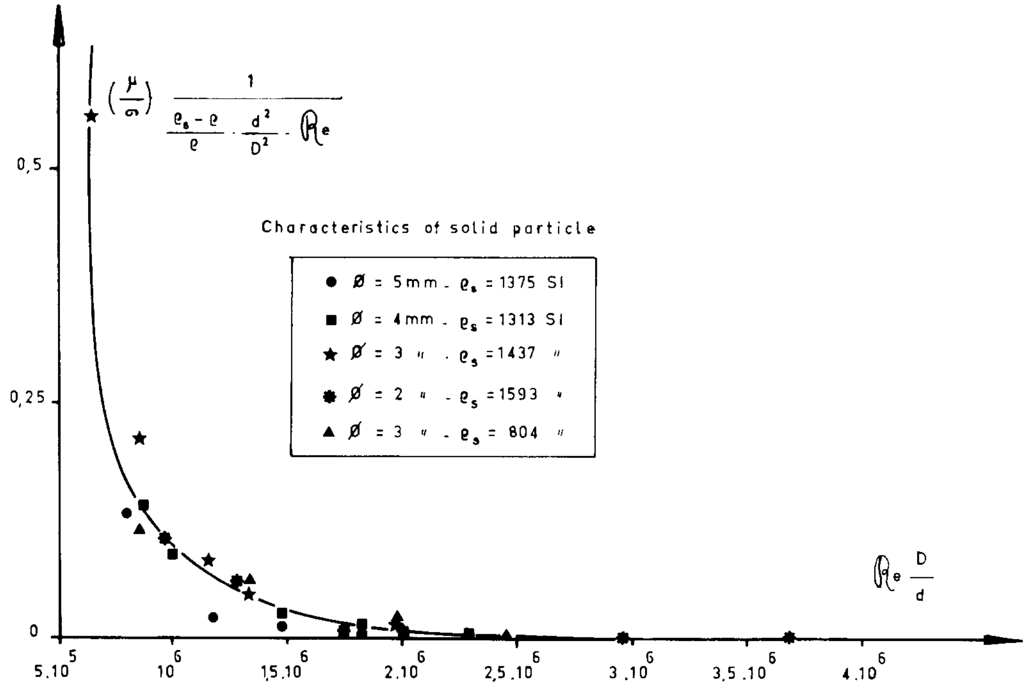


Figure 4. Radial dispersion study—variation of variance of displacement with flow conditions.

parameter ReD/d characterizes the dependence between Λ and d. Figure 4 shows that the radial dispersion, principally due to turbulent fluctuations is in our experiments a function of Λ/d.

5. ANGULAR DISPERSION STUDY

In the case of an inclined pipe, the mean displacement due to the action of gravity is such that we have:

$$\begin{cases} M^r = \mu \cos \theta & \text{by choosing the origin at } \theta = 0 \\ M^\theta = \mu \sin \theta & \text{following the direction } (\rho_s - \rho)\vec{g} \end{cases}$$

$$\mu = \frac{V_{ch} \cos \alpha \, V_{ch} \sin \alpha}{V_m - V_{ch} \sin \alpha} \qquad [4]$$

where V_{ch} is the fall velocity of particles and V_m is the mean velocity of the flow. Using the hypothesis: (i) $\sigma^{r\theta} = \sigma^{\theta r} = 0$; (ii) $\sigma^{rr} = \sigma^{\theta\theta} = \sigma$.

We obtain the equation:

$$\frac{1}{2}\sigma\left\{\partial_{rr}[r \cdot f(r,\theta)] - \left(\partial_{rr} - \frac{1}{r}\partial_\theta\right) f(r,\theta)\right\} - \mu\{\partial_r[r \cos \theta f(r,\theta)] + \mu\partial_\theta[\sin \theta f(r,\theta)]\} = 0.$$

This equation is hardly analytically integrable, we have preferred to use a numerical simulation of the particle displacement in the pipe.

5.1 *Numerical simulation*

This follows nearly the definition of a stochastic process:

We divide the section s into n^2 elements and in each we put initially a constant number of particle. For each particle, we pull out 2 random numbers, BR and $B\theta$, taking place in 2 independent series of random numbers, the mean of which corresponds to the mean radial and mean angular displacement. The particle which was in position $X_0(r_0, \theta_0)$ is now in position $x_1(r_1, \theta_1)$ so that:

$$\begin{cases} r_1 = r_0 + BR \\ \theta_1 = \theta_0 + B\theta/r_0 \end{cases}.$$

When we have displaced every particle, we obtain a new repartition. We repeat the process until we obtain a stable distribution corresponding to the equilibrium distribution of the process.

5.2 *Obtained results*

Associating the results obtained by numerical simulation and the experimental results by identification of second order moments of the angular marginal distribution, which characterizes the angular dispersion, we obtain value of (μ/σ) for each experiment.

Taking into account [4], $(\mu/\sigma) \cdot [(V_m - V_{ch} \sin \alpha)/V_{ch}]$ must vary linearly with $\cos \alpha$ for a given sphere and a given discharge, which is approximately verified. The slope of the straight line obtained is proportional to $(1/\sigma)$. As we can see on figure 5, there is no systematic classification of $(1/\sigma)$ as a function of diameter, so, the solid particles can be a good tracer for the study of the bid eddies.

6. CONCLUSIONS

The results we have obtained show that the solid particle dispersion is due to two phenomena:

—A dispersion due to turbulent velocity fluctuations which, in our experiments, where the particles are of the same order of magnitude as the macroscale of turbulence, is a function of particle dimension.

A dispersion due to eddies of same order of magnitude as the pipe radius, which is not a function of particles dimension.

This latter is certainly influenced by limit conditions of the flow.

It is difficult to characterize in the actual state of our experiments where we have only measured the equilibrium distributions of passage probabilities; however, it seems important to note that it is the essential basis of the dispersion in our experimental condition and it is not

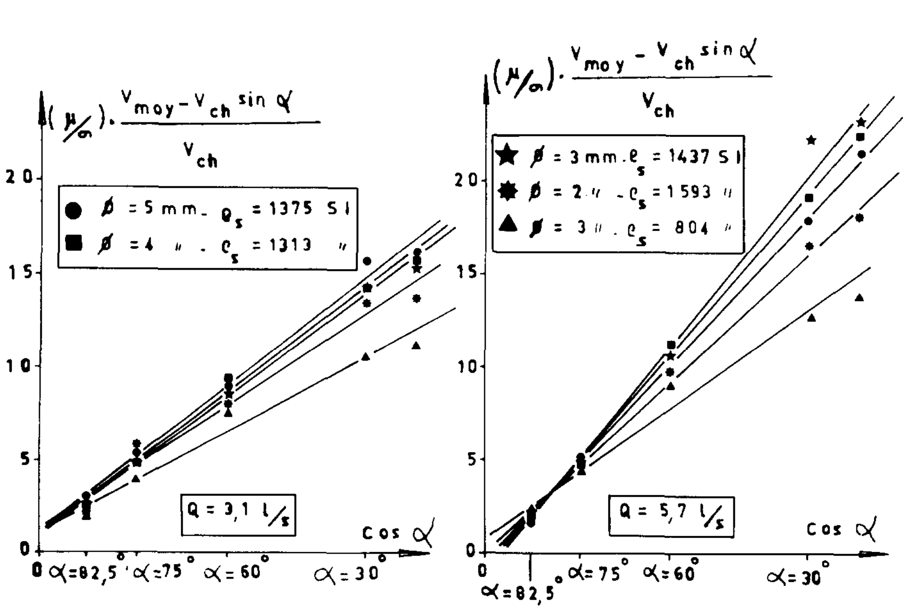


Figure 5.

bad. We have actually done a new series of experiments which can permit us to directly measure the parameter σ by measuring the different terms of passage matrix. These tests are complex because the large number of terms (if n terms are sufficient to measure π, we have n^2 terms to characterize P, and necessitates a modification of the registry system of passage coordinates of a particle.

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